thornado-Hydro (xCFC)

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thornado

https://github.com/endeve/thornado

My Website

https://www.samueljdunham.com

thornado

toolkit for high-order neutrino-radiation hydrodynamics

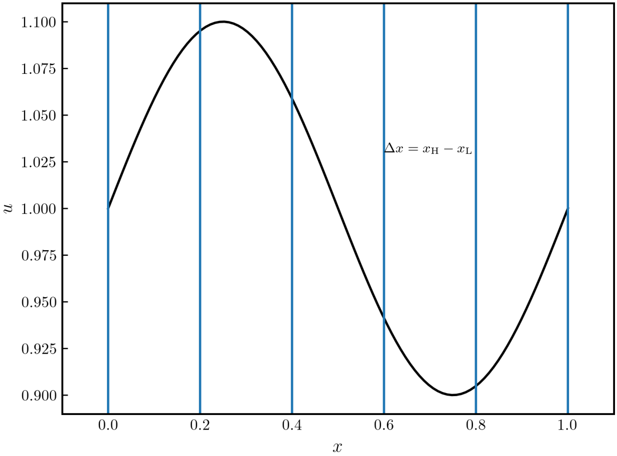
- DG
- SSPRK/IMEX
- GR (xCFC)
- Hydro[a] (Valencia)
- Neutrino transport[b] (M1)
- Interfaces to tabulated EoS/Opacities (weaklib: `https://github.com/starkiller-astro/weaklib`)
- Fluid self-gravity via Poseidon: `https://github.com/jrober50/Poseidon`
- GPUs via OpenACC or OpenMP pragmas
- MPI parallelism and AMR via AMReX: `https://github.com/AMReX-Codes/amrex`

[a] Endeve et al. (2019); Dunham et al. (2020); Pochik et al. (2021)

[b] Laiu et al. (2021)

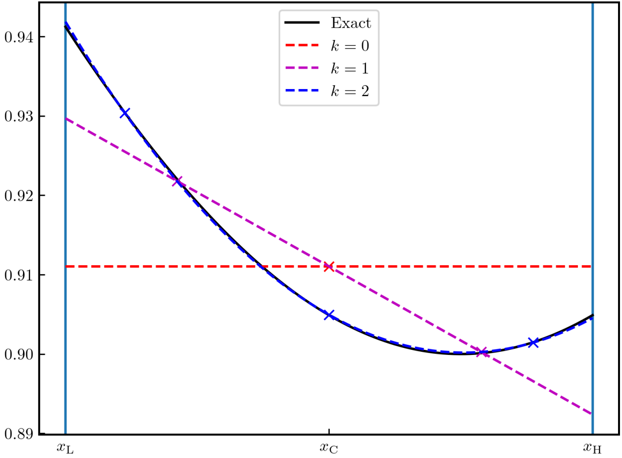
Discontinuous Galerkin (DG) Method

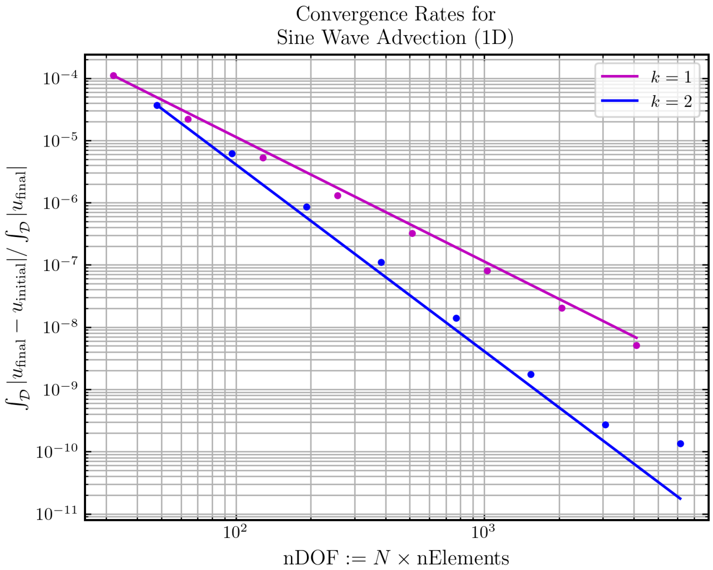
$$u\,(x) = 1 + 0.1\,\sin\,(2\pi x)$$



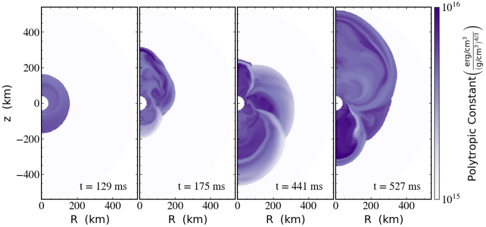
Discontinuous Galerkin (DG) Method

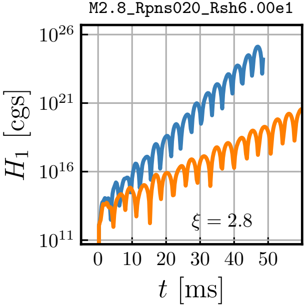
$$u_h\left(x,t\right) := \sum_{i=1}^{k+1} u_i\left(t\right)\,\ell_i\left(x\right)$$




Convergence Rates for
Sine Wave Advection (1D)
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
$\int_{\mathcal{D}} |u_{\text{final}} - u_{\text{initial}}| / \int_{\mathcal{D}} |u_{\text{final}}|$
10^2
10^3
nDOF := $N \times$ nElements
$k = 1$
$k = 2$

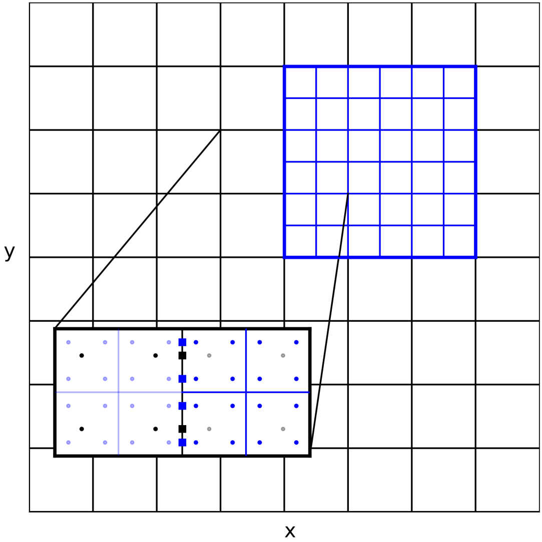
Used `thornado` to investigate the role of GR on the SASI[1]



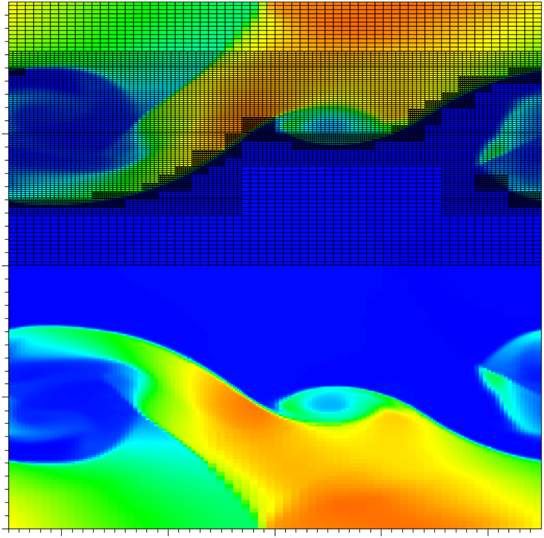


[1] Dunham et al. (2020, 2023)

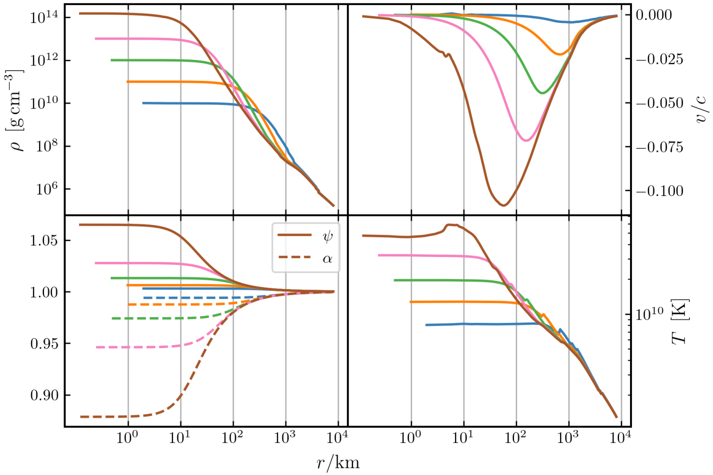
Mesh Refinement



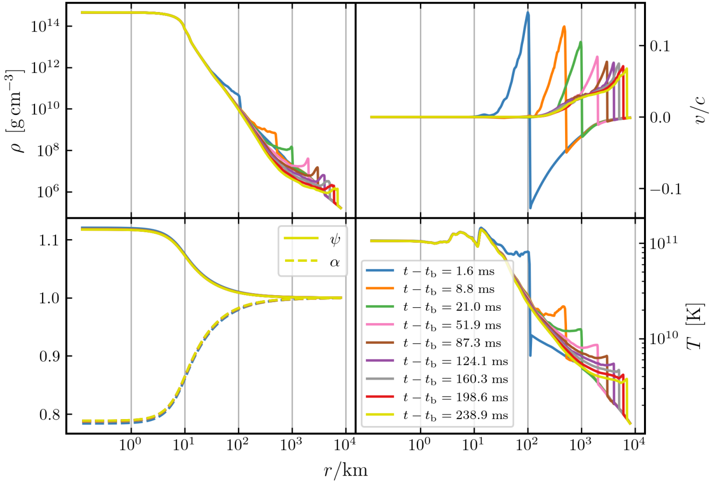
Kelvin–Helmholtz Instability



Adiabatic Collapse (AMR, Collapse Phase)



Adiabatic Collapse (AMR, Post-Bounce Phase)



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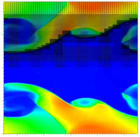
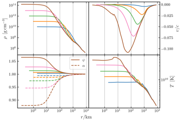
Summary

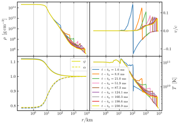
Can run pure hydro problems in GR with AMR



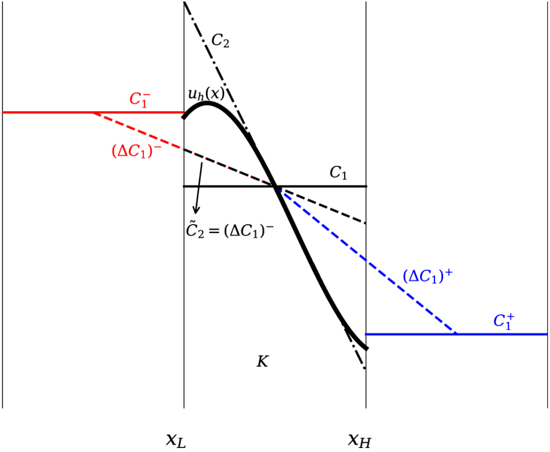


Can run hydro+self-gravity problems in GR with AMR

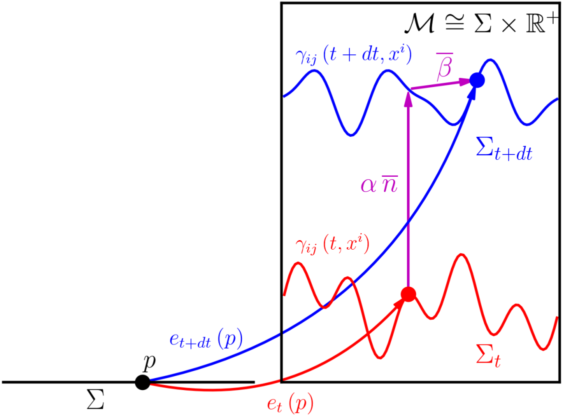
Working on coupling GR transport to existing hydro+gravity modules



$$u_h\left(x,t\right)=\sum_{n=1}^{N}C_n\left(t\right)\ P_n\left(x\right)\implies\tilde{u}_h\left(x,t\right)=C_1\left(t\right)\ P_1\left(x\right)+\tilde{C}_2\left(t\right)\ P_2\left(x\right)$$



3+1 Decomposition



$$ds^2 = g_{\mu\nu}\,dx^\mu\,dx^\nu = -\alpha^2\,dt^2 + \gamma_{ij}\left(dx^i + \beta^i\,dt\right)\left(dx^j + \beta^j\,dt\right)$$

Conformally-Flat Condition

Developed by Wilson et al. (1996), extended by Cordero-Carrión et al. (2009)

$$\gamma_{ij}\left(x\right)=\psi^{4}\left(x\right)\,\overline{\gamma}_{ij}\left(x^{i}\right)$$
$$K=0,\;\partial_{t}\,K=0$$
$$\text{(Always and everywhere)}$$

- Exact in spherical symmetry!
- Hyperbolic $\rightarrow$ Elliptic equations
- Good for long-time simulations

Special case: Schwarzchild spacetime in isotropic coordinates ($G=c=1$)

$$\alpha=\left(1+\frac{1}{2}\,\Phi\right)\left(1-\frac{1}{2}\,\Phi\right)^{-1}$$
$$\psi=1-\frac{1}{2}\,\Phi$$
$$\beta^{i}=0,$$

with

$$\Phi\left(r\right):=-\frac{M}{r}$$