A Comparison of the SASI under Newtonian and General Relativistic Conditions

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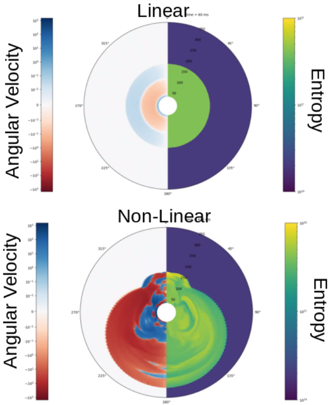
What is the SASI?

Standing Accretion Shock Instability [2]

- After stellar core bounce, shock stalls, still accreting matter
- Accretion shock unstable to non-radial perturbations
- In 2D, instability dominated by $\ell = 1$ mode in Legendre decomposition



Why should you care?

Impacts

- Assists shock revival
- Impacts explosion morphology
- Impacts neutrino emission
- Produces gravitational waves

How does GR affect SASI?

- We perform a parameter study: vary mass of PNS, $M/\mathrm{M}_\odot \in \{1.4, 2.0, 2.8\}$, and initial shock radius, $R_s/\mathrm{km} \in \{120, 150, 180\}$

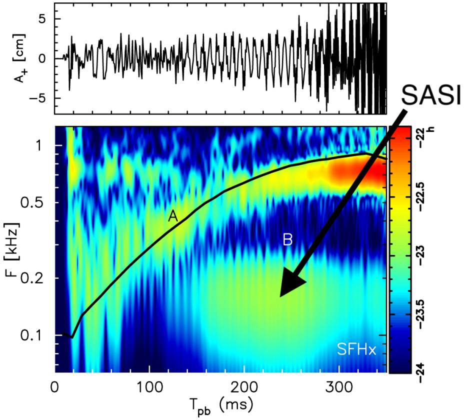


Figure: Spectrogram from simulation of non-rotating $15\,\mathrm{M}_\odot$ progenitor [3].

thornado

toolkit for high-order neutrino radiation hydrodynamics

- Publicly available on GitHub: `https://www.github.com/endeve/thornado`
- Solves (GR)HD equations with dynamic spacetimes
- Solves GR neutrino moment equations (under testing)
- Uses extended conformally-flat condition of GR
- Uses Runge–Kutta discontinuous Galerkin methods
- We compare results of the SASI using Newtonian **and** general relativistic treatments

Initial Conditions

- Stationary metric:
  $$ds^2 = -\alpha^2(r)\, dt^2 + \psi^4(r)\, \overline{\gamma}_{ij}\, dx^i\, dx^j$$
- Spherical PNS of radius 40 km
- Outer boundary of 360 km
- Polytropic process: $p = K\,\rho^{4/3}$
- K fixed above and below shock
- Solve steady state hydro equations:
  $$\frac{1}{\sqrt{\gamma}}\,\partial_r\left[\alpha\,\sqrt{\gamma}\,\boldsymbol{F}(\boldsymbol{U})\right] = \alpha\,\boldsymbol{S}(\boldsymbol{U})$$

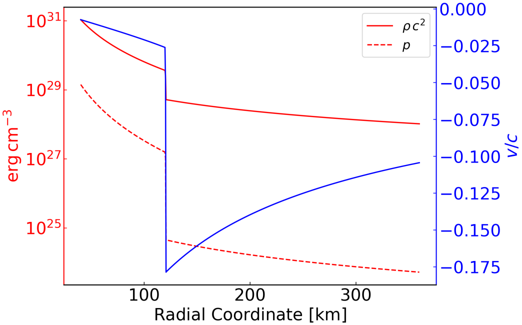


Figure: Initial conditions for model `GR_M1.4_Rs120`.

Initial Conditions

- Evolve in 1D to remove transients
- Map steady solution to 2D
- Slightly perturb post-shock pressure:

$$\Delta p\,(r,\theta) = 10^{-6}\,p\,(r_c)\;e^{\frac{-(r-r_c)^2}{2\,\sigma^2}}\;\cos\theta$$

- Evolve just until non-linear phase is reached ($\sim 300\,\mathrm{ms}$)
- Extract characteristics of SASI in linear regime (growth rate and oscillation period)

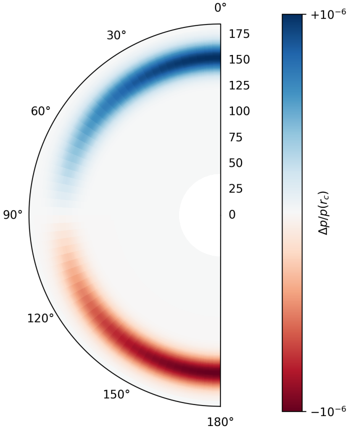


Figure: Pressure perturbation to initiate SASI.

Legendre Decomposition

- $A\left(t,r,\theta\right) := \frac{1}{\sin\theta}\,\partial_\theta\left(v^\theta\left(t,r,\theta\right)\,\sin\theta\right)$ [5]
- Compute Legendre decomposition [1]

  $$G_\ell\left(t,r\right) := \frac{1}{2}\int_0^\pi A\left(t,r,\theta\right)\,P_\ell\left(\cos\theta\right)\,\sin\theta\,d\theta$$

  and power $B_\ell\left(t\right) := \int_{0.8\,R_s}^{0.9\,R_s}\left[G_\ell\left(t,r\right)\right]^2 r^2\,dr$
- Fit $B_1\left(t\right)$ with least-squares to

  $F\left(t\right) := F_1\,e^{2\,\omega_r\,t}\sin^2\left(\omega_i\,t+\delta\right)$ [1]
- Growth rate: $\omega_r = 1/\left(2\,\tau\right)$
- Oscillation frequency: $\omega_i = 2\pi/T$

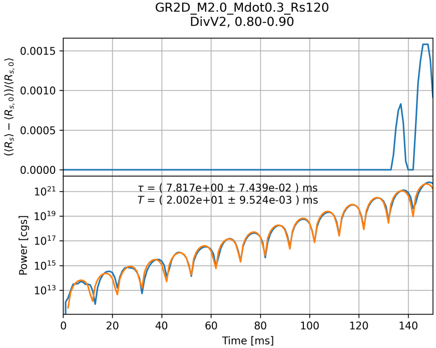


Figure: Top panel: deviation of shock radius from spherical symmetry. Bottom panel: blue line: $B_1\left(t\right)$, orange line: $F\left(t\right)$.

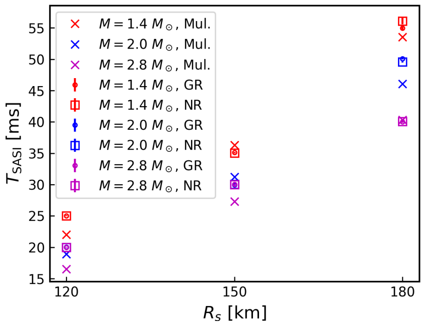
Results: Oscillation Period

- Estimate from Müller (2020) [4]:

$$T_{\mathrm{SASI}} \approx \tau_{\mathrm{ad}} + \tau_{\mathrm{ac}} = \int_{R_{\mathrm{PNS}}}^{R_s} \frac{dr}{|v^r|} + \int_{R_{\mathrm{PNS}}}^{R_s} \frac{dr}{c_s - |v^r|}$$

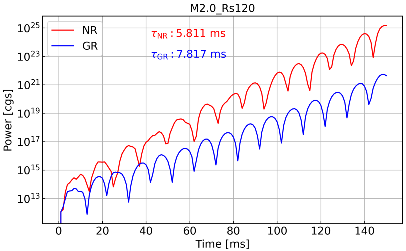
- Good agreement between Newtonian and GR (better than 2%)

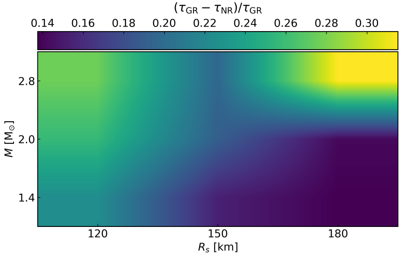
- Both agree well with estimate (within 20%)



Results: Growth Rate

- SASI power in the $\ell = 1$ mode increases faster with Newtonian treatment





Conclusions

Summary

- Within our range of parameters,
  - Newtonian and GR treatments give comparable oscillation periods and agree with estimate
  - GR treatment predicts slower growth rates than Newtonian treatment

Future Work

- Further analysis to understand differences in growth rate
- Vary accretion rate
- Perform study in 3D

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