A Parametric Study of the SASI Comparing General Relativistic and Non-Relativistic Treatments

Samuel J. Dunham

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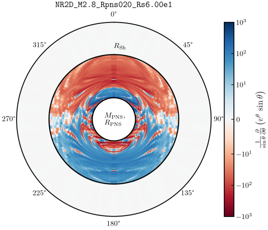
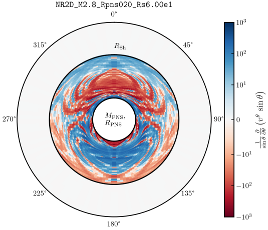
t | $t + 0.25\,T$ | $t + 0.5\,T$


NR2D_M2.8_Rpns020_Rs6.00e1
0°
45°
90°
135°
180°
225°
270°
315°
R_{Sh}
M_{PNS}, R_{PNS}
10^3
10^2
10^1
0
-10^1
-10^2
-10^3
$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(v^\theta \sin\theta\right)$


NR2D_M2.8_Rpns020_Rs6.00e1
0°
45°
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R_{Sh}
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10^3
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10^1
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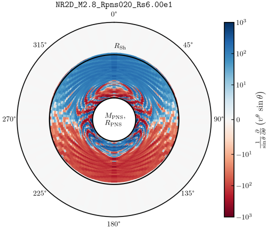
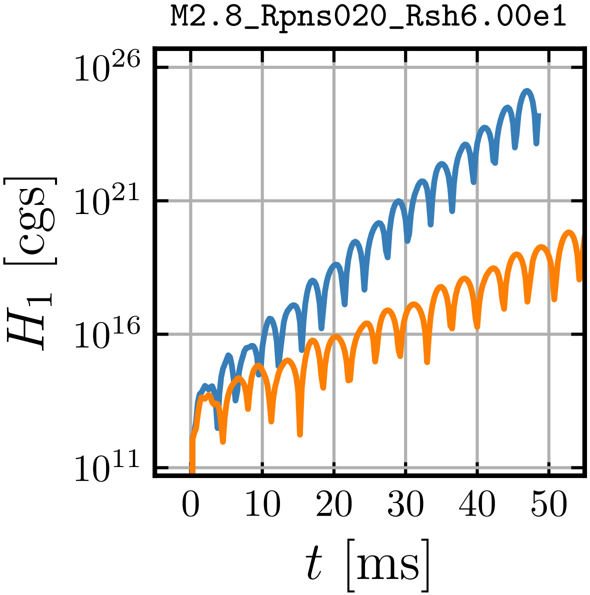
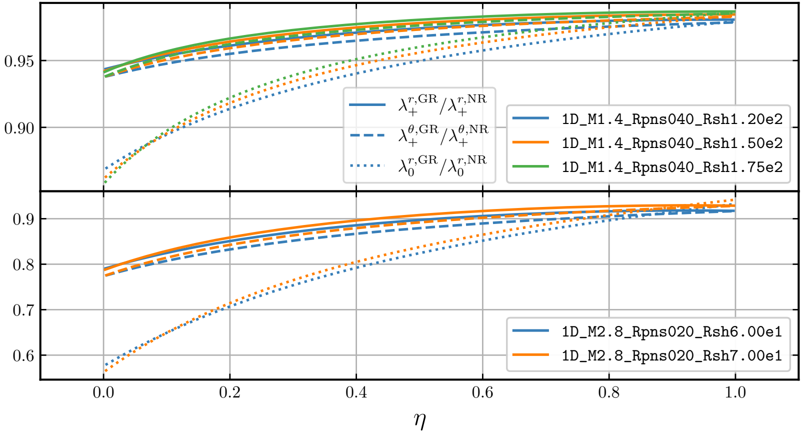
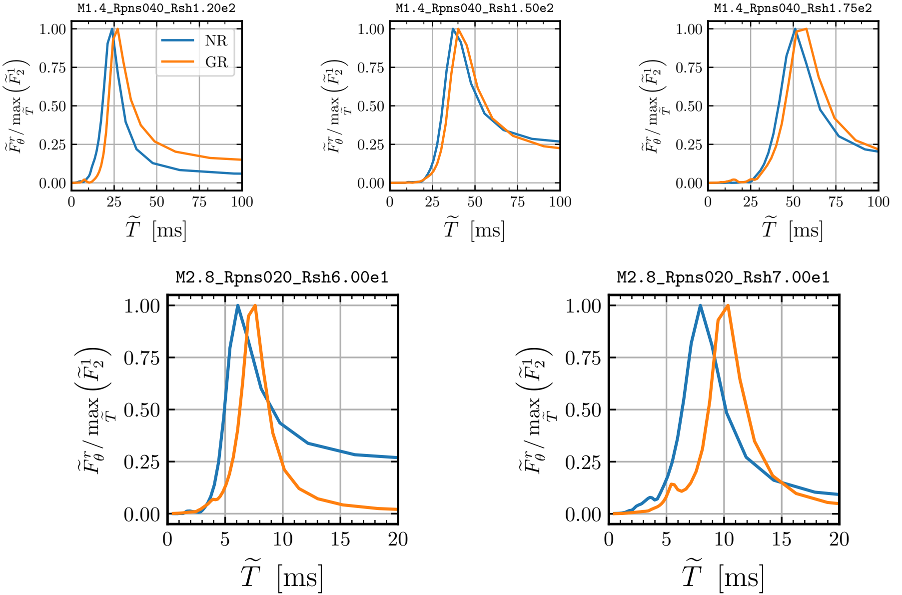
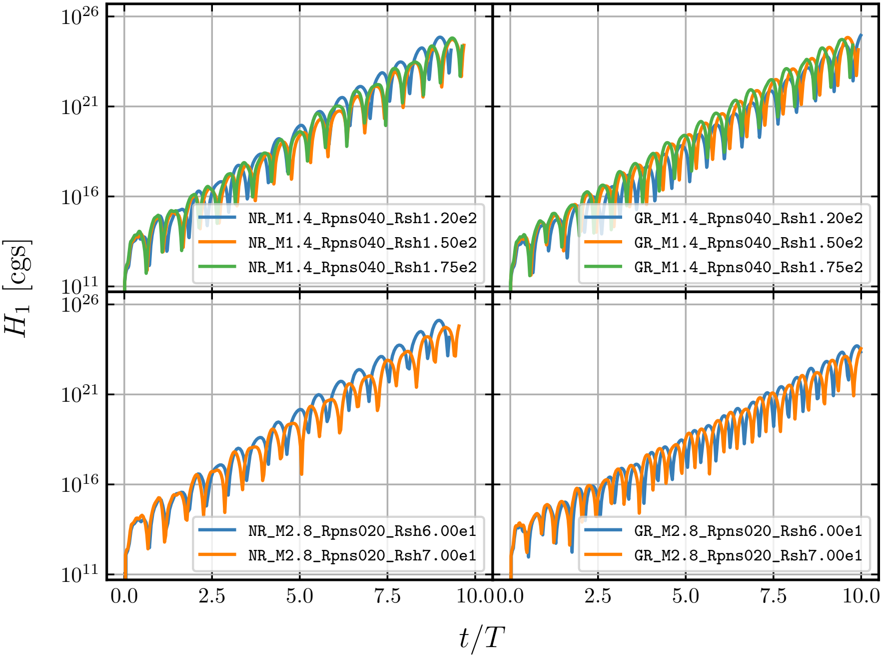
-10^1
-10^2
-10^3
$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(v^\theta \sin\theta\right)$


NR2D_M2.8_Rpns020_Rs6.00e1
0°
45°
90°
135°
180°
225°
270°
315°
R_{Sh}
M_{PNS}, R_{PNS}
10^3
10^2
10^1
0
-10^1
-10^2
-10^3
$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(v^\theta \sin\theta\right)$


M2.8_Rpns020_Rsh6.00e1
10^{26}
10^{21}
10^{16}
10^{11}
H_1 [cgs]
0
10
20
30
40
50
t [ms]

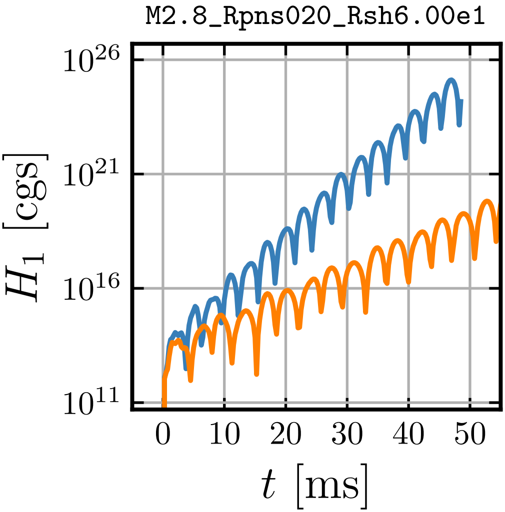

0.95
0.90
0.9
0.8
0.7
0.6
0.0
0.2
0.4
0.6
0.8
1.0
η
$\lambda_+^{r,\mathrm{GR}}/\lambda_+^{r,\mathrm{NR}}$
$\lambda_+^{\theta,\mathrm{GR}}/\lambda_+^{\theta,\mathrm{NR}}$
$\lambda_0^{r,\mathrm{GR}}/\lambda_0^{r,\mathrm{NR}}$
1D_M1.4_Rpns040_Rsh1.20e2
1D_M1.4_Rpns040_Rsh1.50e2
1D_M1.4_Rpns040_Rsh1.75e2
1D_M2.8_Rpns020_Rsh6.00e1
1D_M2.8_Rpns020_Rsh7.00e1


M1.4_Rpns040_Rsh1.20e2
M1.4_Rpns040_Rsh1.50e2
M1.4_Rpns040_Rsh1.75e2
M2.8_Rpns020_Rsh6.00e1
M2.8_Rpns020_Rsh7.00e1
NR
GR
$\widetilde{F}^r_\theta / \max_{\widetilde{T}} \left(\widetilde{F}^1_2\right)$
$\widetilde{T}$ [ms]
0.00
0.25
0.50
0.75
1.00
0
25
50
75
100
5
10
15
20

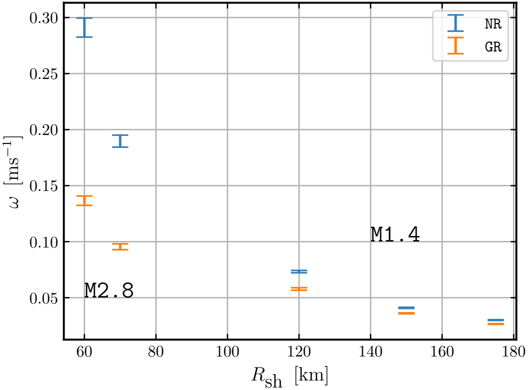


10^{26}
10^{21}
10^{16}
10^{11}
H_1 [cgs]
NR_M1.4_Rpns040_Rsh1.20e2
NR_M1.4_Rpns040_Rsh1.50e2
NR_M1.4_Rpns040_Rsh1.75e2
GR_M1.4_Rpns040_Rsh1.20e2
GR_M1.4_Rpns040_Rsh1.50e2
GR_M1.4_Rpns040_Rsh1.75e2
NR_M2.8_Rpns020_Rsh6.00e1
NR_M2.8_Rpns020_Rsh7.00e1
GR_M2.8_Rpns020_Rsh6.00e1
GR_M2.8_Rpns020_Rsh7.00e1
0.0 2.5 5.0 7.5 10.0
t/T

Conclusions



Conclusions



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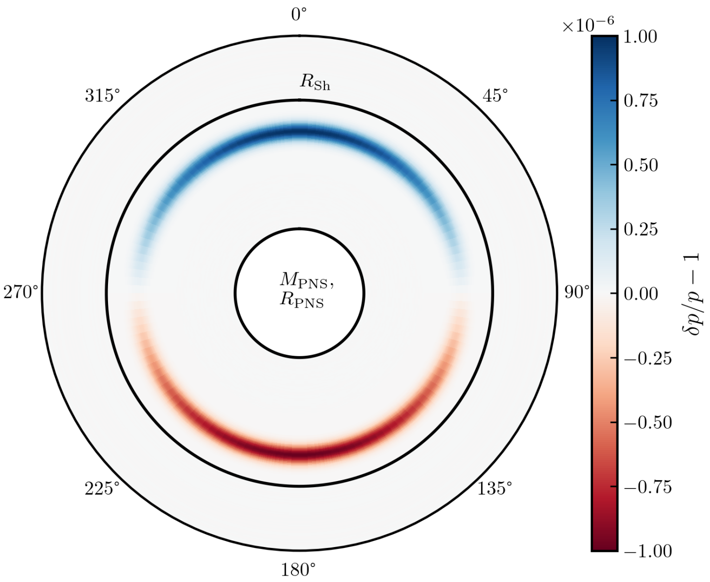
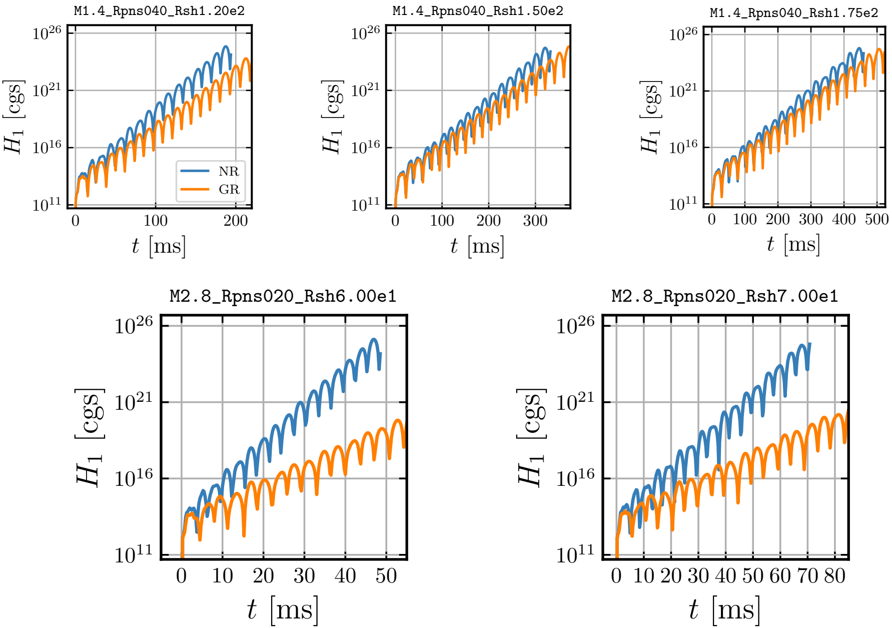
Summary

- Extended study of Blondin and Mezzacappa (2006) to include GR
- Showed that GR leads to longer SASI oscillation period than NR
- Showed that GR leads to smaller SASI growth rate than NR
- Found that growth rate is such that ωT is roughly constant for some parameter sets: implications for growth rate mechanism
- Future Work
  - Extend study to 3D
  - Include GR monopole (Marek et al., 2006)


0°
45°
90°
135°
180°
225°
270°
315°
$R_{\rm Sh}$
$M_{\rm PNS}$,
$R_{\rm PNS}$
$\times 10^{-6}$
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
$\delta p/p - 1$

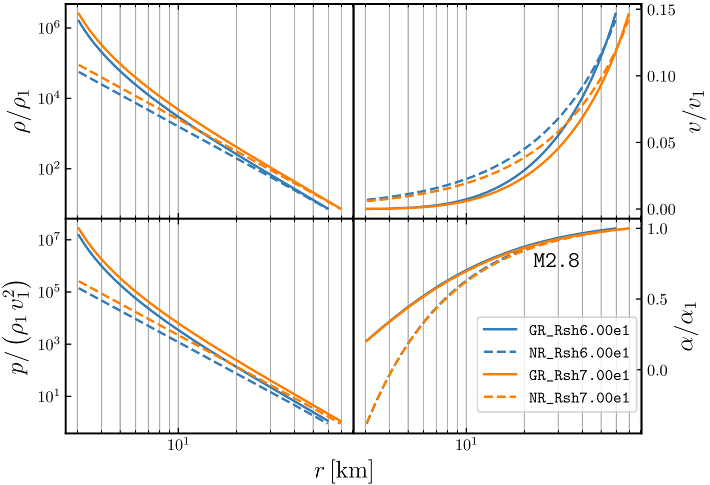

M1.4_Rpns040_Rsh1.20e2
M1.4_Rpns040_Rsh1.50e2
M1.4_Rpns040_Rsh1.75e2
M2.8_Rpns020_Rsh6.00e1
M2.8_Rpns020_Rsh7.00e1
H_1 [cgs]
t [ms]
NR
GR

$$A := \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(v^{\theta}\,\sin\theta\right)$$ (Scheck et al., 2008)

$$A\left(r,\theta,t\right) = \sum_{\ell'=0}^{\infty} G_{\ell'}\left(r,t\right) P_{\ell'}\left(\cos\theta\right)$$

$$\implies G_{\ell}\left(r,t\right) := \frac{1}{N_{\ell}}\int_{0}^{\pi} A\left(r,\theta,t\right)\, P_{\ell}\left(\cos\theta\right)\,\sin\theta\,d\theta$$

$$H_{\ell}\left(t\right) := 4\pi\int_{r_a}^{r_b}\left[G_{\ell}\left(r,t\right)\right]^2\left[\psi\left(r\right)\right]^6 r^2\,dr$$



ρ/ρ_1
10^6
10^4
10^2
v/v_1
0.15
0.10
0.05
0.00
$p/\left(\rho_1 v_1^2\right)$
10^7
10^5
10^3
10^1
α/α_1
1.0
0.5
0.0
M2.8
GR_Rsh6.00e1
NR_Rsh6.00e1
GR_Rsh7.00e1
NR_Rsh7.00e1
10^1
10^1
r [km]

Parameters we varied:
$\xi = (M_{\text{PNS}}/M_{\odot})\,/\,(R_{\text{PNS}}/20\,\text{km})$ (O'Connor and Ott, 2011), $R_{\text{Sh}}\,(t=0)$

Table 1. Model Parameters

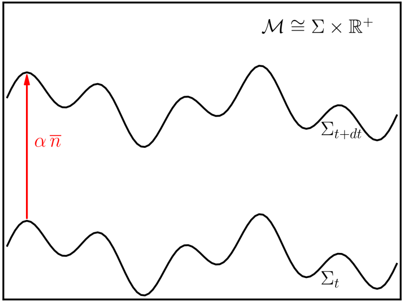
Model	M_{PNS} [$M_{\odot}$]	R_{PNS} [km]	R_{sh} [km]	ξ
`M1.4_Rpns040_Rsh1.20e2`	1.4	40	120	0.7
`M1.4_Rpns040_Rsh1.50e2`	1.4	40	150	0.7
`M1.4_Rpns040_Rsh1.75e2`	1.4	40	175	0.7
`M2.8_Rpns020_Rsh6.00e1`	2.8	20	60	2.8
`M2.8_Rpns020_Rsh7.00e1`	2.8	20	70	2.8

NOTE—Model parameters chosen for the 5 models. All models were run with both GR and NR. The first three rows correspond to the low-compactness models and the last two rows correspond to the high-compactness models.

Conformally-Flat Condition

Wilson et al. (1996); Cordero-Carrión et al. (2009)

$$ds^2 = -\alpha^2\, dt^2 + \gamma_{ij}\, dx^i\, dx^j$$



Conformally-Flat Condition

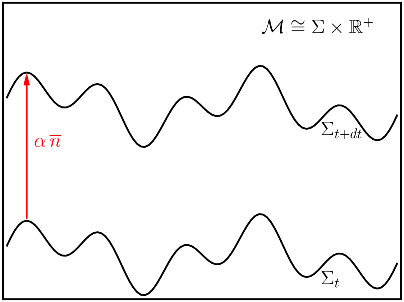
Wilson et al. (1996); Cordero-Carrión et al. (2009)

$$ds^2 = -\alpha^2\, dt^2 + \gamma_{ij}\, dx^i\, dx^j$$

$$\gamma_{ij} = \psi^4\, \bar{\gamma}_{ij}$$

$1 \leq \psi < 2$: Conformal factor

$$\bar{\gamma}_{ij} = \mathrm{diag}\left(1, r^2, r^2 \sin^2\theta\right)$$



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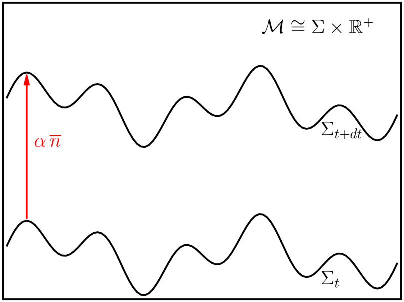
(Also, maximum slicing condition:
$K := \mathrm{Tr}_{\gamma_{ij}}\left(\underline{\underline{K}}\right) = \partial_t K = 0$)



Isotropic Coordinates ($c = G = 1$)

Baumgarte and Shapiro (2010)

$$\alpha\left(r\right) = \left(1 - \frac{R_{\mathrm{Sc}}}{r}\right)\left(1 + \frac{R_{\mathrm{Sc}}}{r}\right)^{-1}$$

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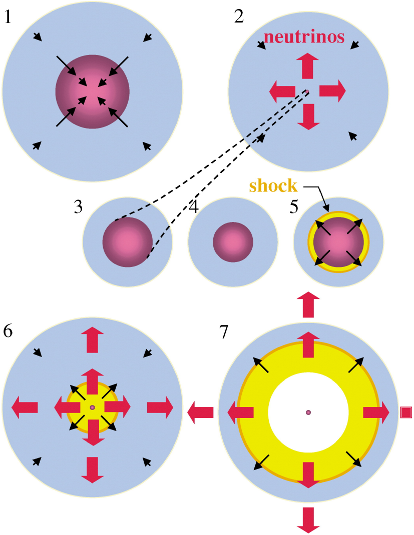
$$\beta^i = 0$$

$$K_{ij} = 0$$



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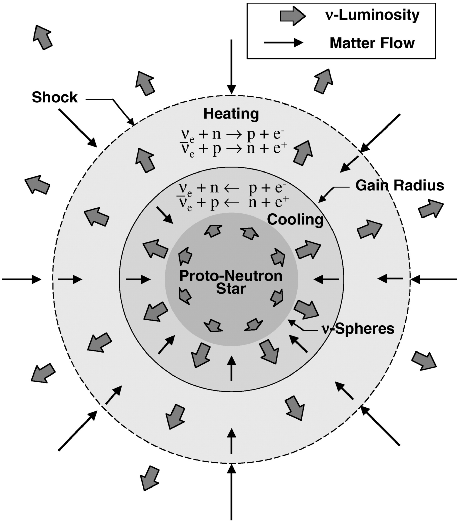

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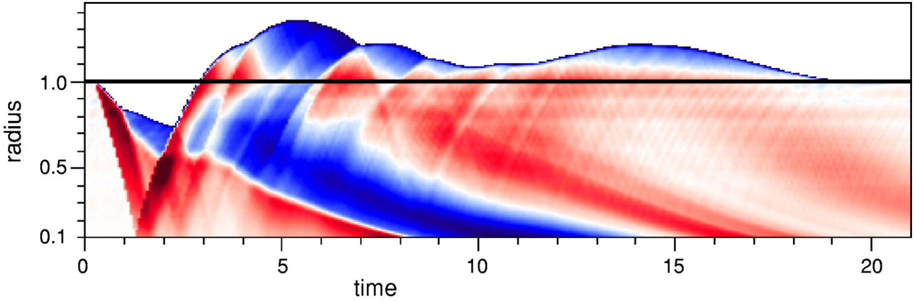


Figure: Equilibrium (white), under- (blue), and over- (red) pressure (Blondin et al., 2003).

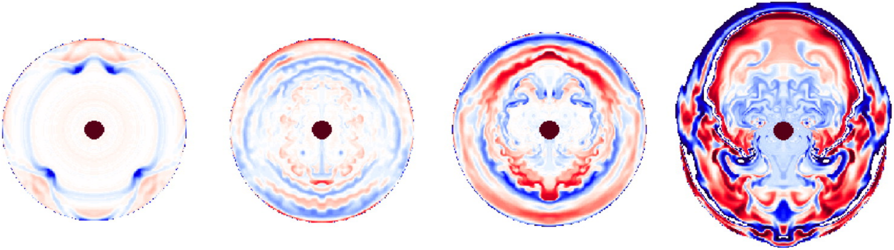
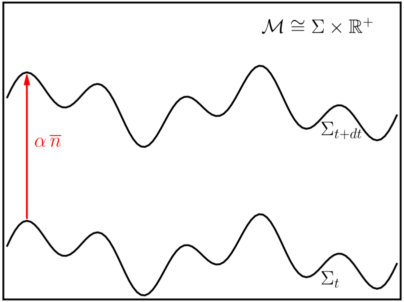


Figure: Equilibrium (white), under- (blue), and over- (red) entropy (Blondin et al., 2003).

$d+1$ Decomposition

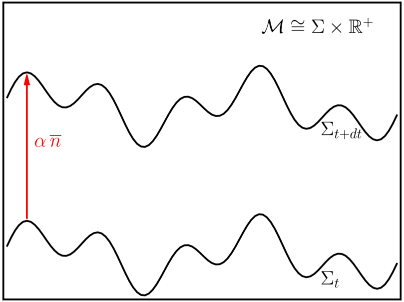
$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$$



$d+1$ Decomposition

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$$

$\underline{\underline{g}}$: spacetime metric on $\mathcal{M}$



d+1 Decomposition

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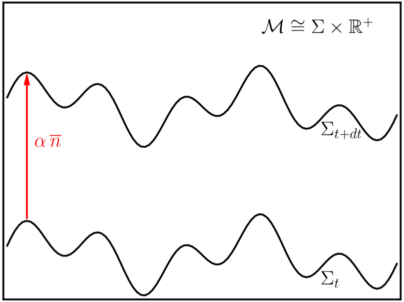
$\underline{\underline{g}}$: spacetime metric on $\mathcal{M}$

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \beta^k \beta_k & \beta_j \\ \beta_i & \gamma_{ij} \end{pmatrix}$$



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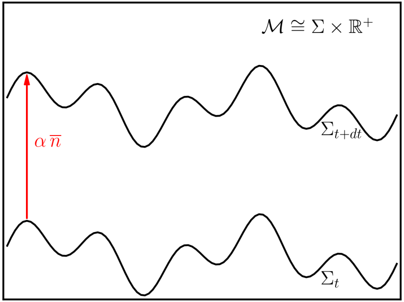
$\underline{\underline{\gamma}}$: spatial three-metric on Σ_t



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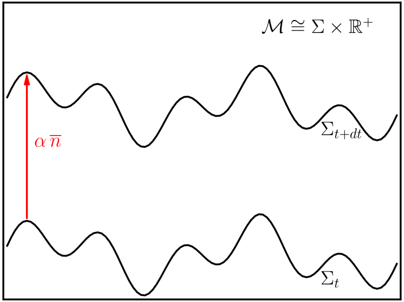
$0 < \alpha \leq 1$: Lapse Function



$d+1$ Decomposition

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$$

$\underline{\underline{g}}$: spacetime metric on $\mathcal{M}$

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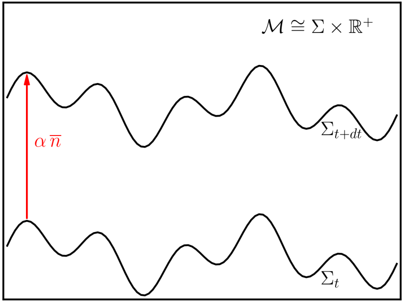
$\overline{n}$: Eulerian four-velocity

$(\underline{\underline{g}}\,(\overline{n}, \overline{n}) = \overline{n} \cdot \overline{n} = -1)$



$d+1$ Decomposition

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$$

$\underline{\underline{g}}$: spacetime metric on $\mathcal{M}$

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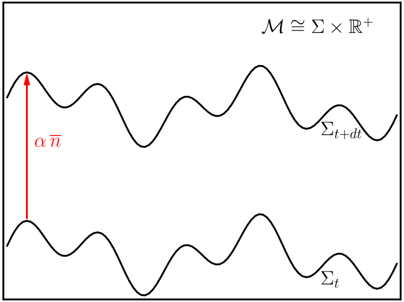
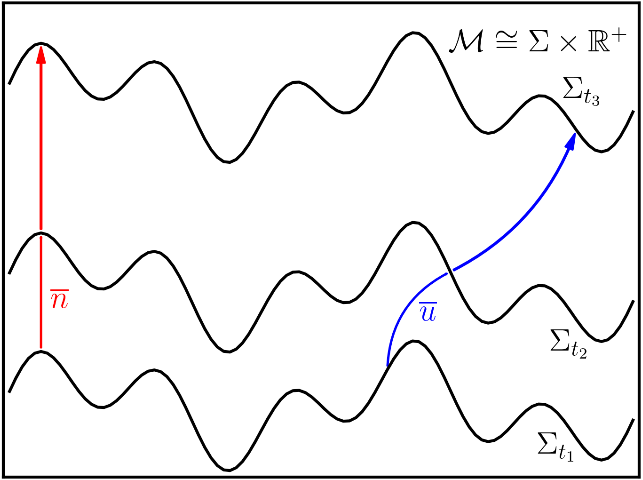
$0 < \alpha \leq 1$: Lapse Function

$\overline{n}$: Eulerian four-velocity

$(\underline{\underline{g}}\,(\overline{n}, \overline{n}) = \overline{n} \cdot \overline{n} = -1)$

(Also, $\underline{\underline{K}}$: Extrinsic curvature)




$\mathcal{M} \cong \Sigma \times \mathbb{R}^+$
Σ_{t_3}
$\overline{n}$
$\overline{u}$
Σ_{t_2}
Σ_{t_1}

Fluid Equations

Units defined such that $c = G = 1$

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$\overline{\nabla} \cdot \overline{J} = 0$ ($\overline{J}$: baryon mass density current four-vector)

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$\overline{\nabla} \cdot \overline{\overline{T}} = \overline{0}$ ($\overline{\overline{T}}$: Rank $(2, 0)$ stress-energy tensor)

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$\overline{\nabla} \cdot \overline{J} = 0$ ($\overline{J}$: baryon mass density current four-vector)

$\overline{\nabla} \cdot \overline{\overline{T}} = \overline{0}$ ($\overline{\overline{T}}$: Rank $(2,0)$ stress-energy tensor)

$\overline{J} := \rho\,\overline{u}$ (ρ: comoving baryon mass density)

Fluid Equations

Units defined such that $c = G = 1$

$\overline{\nabla} \cdot \overline{J} = 0$ ($\overline{J}$: baryon mass density current four-vector)

$\overline{\nabla} \cdot \overline{\overline{T}} = \overline{0}$ ($\overline{\overline{T}}$: Rank $(2,0)$ stress-energy tensor)

$\overline{J} := \rho\,\overline{u}$ (ρ: comoving baryon mass density)

$\overline{\overline{T}} := \rho\,h\,\overline{u} \otimes \overline{u} + p\,\overline{\overline{g}}$ (p: comoving thermal pressure,
$h := 1 + (e + p)\,/\rho$: specific enthalpy, e: comoving internal energy density)

Fluid Equations

Units defined such that $c = G = 1$

$\overline{\nabla} \cdot \overline{J} = 0$ ($\overline{J}$: baryon mass density current four-vector)
$\overline{\nabla} \cdot \overline{\overline{T}} = \overline{0}$ ($\overline{\overline{T}}$: Rank $(2,0)$ stress-energy tensor)

$\overline{J} := \rho\,\overline{u}$ (ρ: comoving baryon mass density)

$\overline{\overline{T}} := \rho\,h\,\overline{u} \otimes \overline{u} + p\,\overline{\overline{g}}$ (p: comoving thermal pressure,
$h := 1 + (e + p)\,/\rho$: specific enthalpy, e: comoving internal energy density)

Five equations with six unknowns ☹

Fluid Equations

Units defined such that $c = G = 1$

$\overline{\nabla} \cdot \overline{J} = 0$ ($\overline{J}$: baryon mass density current four-vector)

$\overline{\nabla} \cdot \overline{\overline{T}} = \overline{0}$ ($\overline{\overline{T}}$: Rank $(2, 0)$ stress-energy tensor)

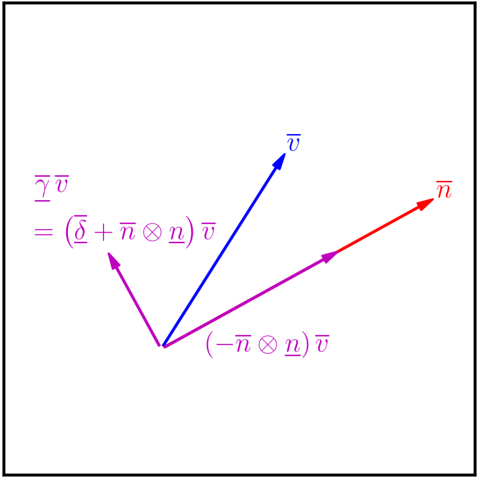
$\overline{J} := \rho \, \overline{u}$ (ρ: comoving baryon mass density)

$\overline{\overline{T}} := \rho \, h \, \overline{u} \otimes \overline{u} + p \, \overline{\overline{g}}$ (p: comoving thermal pressure,
$h := 1 + (e + p) / \rho$: specific enthalpy, e: comoving internal energy density)

Five equations with six unknowns ☹

Close with an equation of state: $p = p(e) := (\Gamma - 1) \, e$, $\Gamma = 4/3$

Valencia Decomposition



Extensible to higher-rank tensors!

Valencia Decomposition

$$E := n_{\mu'} \, n_{\nu'} \, T^{\mu' \nu'}$$

Valencia Decomposition

$$E := n_{\mu'}\, n_{\nu'}\, T^{\mu'\nu'}$$

$$S^{\mu} := -\gamma^{\mu}{}_{\mu'}\, n_{\nu'}\, T^{\mu'\nu'}$$

Valencia Decomposition

$$E := n_{\mu'} \, n_{\nu'} \, T^{\mu' \nu'}$$

$$S^{\mu} := -\gamma^{\mu}{}_{\mu'} \, n_{\nu'} \, T^{\mu' \nu'}$$

$$P^{\mu\nu} := \gamma^{\mu}{}_{\mu'} \, \gamma^{\nu}{}_{\nu'} \, T^{\mu' \nu'}$$

Valencia Decomposition

$$E := n_{\mu'}\, n_{\nu'}\, T^{\mu'\nu'}$$

$$S^{\mu} := -\gamma^{\mu}{}_{\mu'}\, n_{\nu'}\, T^{\mu'\nu'}$$

$$P^{\mu\nu} := \gamma^{\mu}{}_{\mu'}\, \gamma^{\nu}{}_{\nu'}\, T^{\mu'\nu'}$$

Math...

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma}\, \boldsymbol{F}^i(\boldsymbol{U})\right] = \boldsymbol{S}(\boldsymbol{U})$$

GR

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma} \boldsymbol{F}^i (\boldsymbol{U}) \right] = \boldsymbol{S} (\boldsymbol{U})$$

GR

$\boldsymbol{U} := (D, S_j, \tau)^\top$

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}}\,\partial_i\left[\alpha\,\sqrt{\gamma}\,\boldsymbol{F}^i\left(\boldsymbol{U}\right)\right] = \boldsymbol{S}\left(\boldsymbol{U}\right)$$

GR

$\boldsymbol{U} := \left(D, S_j, \tau\right)^\top$

$D := \rho\,W$

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma} \boldsymbol{F}^i (\boldsymbol{U}) \right] = \boldsymbol{S} (\boldsymbol{U})$$

GR

$\boldsymbol{U} := (D, S_j, \tau)^\top$

$D := \rho W$

$S_j := \rho h W^2 v_j$

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma} \boldsymbol{F}^i (\boldsymbol{U})\right] = \boldsymbol{S} (\boldsymbol{U})$$

GR

$\boldsymbol{U} := (D, S_j, \tau)^\top$

$D := \rho W$

$S_j := \rho h W^2 v_j$

$\tau := E - D = \rho h W^2 - p - \rho W$

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma} \boldsymbol{F}^i (\boldsymbol{U})\right] = \boldsymbol{S}(\boldsymbol{U})$$

GR

$\boldsymbol{U} := (D, S_j, \tau)^\top$

$D := \rho W$

$S_j := \rho h W^2 v_j$

$\tau := E - D = \rho h W^2 - p - \rho W$

$\alpha = (1 - R_{\mathrm{Sc}}/r) / (1 + R_{\mathrm{Sc}}/r)$

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma} \boldsymbol{F}^i \left(\boldsymbol{U} \right) \right] = \boldsymbol{S} \left(\boldsymbol{U} \right)$$

GR

$\boldsymbol{U} := \left(D, S_j, \tau \right)^\top$

$D := \rho W$

$S_j := \rho h W^2 v_j$

$\tau := E - D = \rho h W^2 - p - \rho W$

$\alpha = \left(1 - R_{\text{Sc}}/r \right) / \left(1 + R_{\text{Sc}}/r \right)$

$\sqrt{\gamma} = \psi^6 \sqrt{\bar{\gamma}}$

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma} \boldsymbol{F}^i (\boldsymbol{U})\right] = \boldsymbol{S}(\boldsymbol{U})$$

GR

NR

$\boldsymbol{U} := (D, S_j, \tau)^\top$

$D := \rho W$

$S_j := \rho h W^2 v_j$

$\tau := E - D = \rho h W^2 - p - \rho W$

$\alpha = (1 - R_{\text{Sc}}/r) / (1 + R_{\text{Sc}}/r)$

$\sqrt{\gamma} = \psi^6 \sqrt{\bar{\gamma}}$

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma} \boldsymbol{F}^i (\boldsymbol{U})\right] = \boldsymbol{S}(\boldsymbol{U})$$

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NR

$\boldsymbol{U} := (D, S_j, \tau)^\top$

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma} \boldsymbol{F}^i \left(\boldsymbol{U}\right)\right] = \boldsymbol{S}\left(\boldsymbol{U}\right)$$

GR

$\boldsymbol{U} := \left(D, S_j, \tau\right)^\top$

$D := \rho W$

$S_j := \rho h W^2 v_j$

$\tau := E - D = \rho h W^2 - p - \rho W$

$\alpha = \left(1 - R_{\mathrm{Sc}}/r\right) / \left(1 + R_{\mathrm{Sc}}/r\right)$

$\sqrt{\gamma} = \psi^6 \sqrt{\bar{\gamma}}$

NR

$\boldsymbol{U} := \left(D, S_j, \tau\right)^\top$

$D := \rho$

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma} \, \boldsymbol{F}^i \left(\boldsymbol{U} \right) \right] = \boldsymbol{S} \left(\boldsymbol{U} \right)$$

GR

$\boldsymbol{U} := \left(D, S_j, \tau \right)^\top$

$D := \rho \, W$

$S_j := \rho \, h \, W^2 \, v_j$

$\tau := E - D = \rho \, h \, W^2 - p - \rho \, W$

$\alpha = \left(1 - R_{\mathrm{Sc}} / r \right) / \left(1 + R_{\mathrm{Sc}} / r \right)$

$\sqrt{\gamma} = \psi^6 \sqrt{\bar{\gamma}}$

NR

$\boldsymbol{U} := \left(D, S_j, \tau \right)^\top$

$D := \rho$

$S_j := \rho \, v_j$

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma} \boldsymbol{F}^i (\boldsymbol{U})\right] = \boldsymbol{S}(\boldsymbol{U})$$

GR

$\boldsymbol{U} := (D, S_j, \tau)^\top$

$D := \rho W$

$S_j := \rho h W^2 v_j$

$\tau := E - D = \rho h W^2 - p - \rho W$

$\alpha = (1 - R_{\mathrm{Sc}}/r) / (1 + R_{\mathrm{Sc}}/r)$

$\sqrt{\gamma} = \psi^6 \sqrt{\bar{\gamma}}$

NR

$\boldsymbol{U} := (D, S_j, \tau)^\top$

$D := \rho$

$S_j := \rho v_j$

$\tau = e + \frac{1}{2} \rho v^i v_i$

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma} \boldsymbol{F}^i (\boldsymbol{U})\right] = \boldsymbol{S} (\boldsymbol{U})$$

GR

$\boldsymbol{U} := (D, S_j, \tau)^\top$

$D := \rho W$

$S_j := \rho h W^2 v_j$

$\tau := E - D = \rho h W^2 - p - \rho W$

$\alpha = (1 - R_{\mathrm{Sc}}/r) / (1 + R_{\mathrm{Sc}}/r)$

$\sqrt{\gamma} = \psi^6 \sqrt{\bar{\gamma}}$

NR

$\boldsymbol{U} := (D, S_j, \tau)^\top$

$D := \rho$

$S_j := \rho v_j$

$\tau = e + \frac{1}{2} \rho v^i v_i$

$\alpha = 1$

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma} \boldsymbol{F}^i (\boldsymbol{U}) \right] = \boldsymbol{S} (\boldsymbol{U})$$

GR

$\boldsymbol{U} := (D, S_j, \tau)^\top$

$D := \rho W$

$S_j := \rho h W^2 v_j$

$\tau := E - D = \rho h W^2 - p - \rho W$

$\alpha = (1 - R_{\mathrm{Sc}}/r) / (1 + R_{\mathrm{Sc}}/r)$

$\sqrt{\gamma} = \psi^6 \sqrt{\bar{\gamma}}$

NR

$\boldsymbol{U} := (D, S_j, \tau)^\top$

$D := \rho$

$S_j := \rho v_j$

$\tau = e + \frac{1}{2} \rho v^i v_i$

$\alpha = 1$

$\sqrt{\gamma} = \sqrt{\bar{\gamma}}$

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma} \, \boldsymbol{F}^i (\boldsymbol{U}) \right] = \boldsymbol{S} (\boldsymbol{U})$$

GR

$$\boldsymbol{F}^i (\boldsymbol{U}) = \begin{pmatrix} \rho \, W \, v^i \\ \rho \, h \, W^2 \, v^i \, v_j + p \, \delta^i{}_j \\ \left(\rho \, h \, W^2 - \rho \, W \right) \, v^i \end{pmatrix}$$

NR

$$\boldsymbol{F}^i (\boldsymbol{U}) = \begin{pmatrix} \rho \, v^i \\ \rho \, v^i \, v_j + p \, \delta^i{}_j \\ \left(\rho \, h_{\mathrm{NR}} + \frac{1}{2} \, v^j \, v_j \right) \, v^i \end{pmatrix}$$

Valencia Decomposition

$$\partial_t \boldsymbol{U} + \frac{1}{\sqrt{\gamma}} \partial_i \left[\alpha \sqrt{\gamma} \, \boldsymbol{F}^i \left(\boldsymbol{U} \right) \right] = \boldsymbol{S} \left(\boldsymbol{U} \right)$$

GR

$$\boldsymbol{S} \left(\boldsymbol{U} \right) = \begin{pmatrix} 0 \\ \frac{1}{2} \, \alpha \, P^{ik} \, \partial_j \, \gamma_{ik} - E \, \partial_j \, \alpha \\ -S^j \, \partial_j \, \alpha \end{pmatrix}$$

NR

$$\boldsymbol{S} \left(\boldsymbol{U} \right) = \begin{pmatrix} 0 \\ \frac{1}{2} \, P^{ik} \, \partial_j \, \bar{\gamma}_{ik} - \rho \, \partial_j \, \Phi \\ -S^j \, \partial_j \, \Phi \end{pmatrix}$$

$$\Phi \left(r \right) := -M/r$$

Steady-State Solutions (pre-shock)

$$\cancelto{0}{\partial_t \boldsymbol{U}} + \frac{1}{\sqrt{\gamma}} \partial_r \left[\alpha \sqrt{\gamma} \, \boldsymbol{F}^r \left(\boldsymbol{U} \right) \right] = \boldsymbol{S} \left(\boldsymbol{U} \right)$$

GR

Steady-State Solutions (pre-shock)

$$\cancelto{0}{\partial_t \boldsymbol{U}} + \frac{1}{\sqrt{\gamma}}\,\partial_r\left[\alpha\,\sqrt{\gamma}\,\boldsymbol{F}^r\left(\boldsymbol{U}\right)\right] = \boldsymbol{S}\left(\boldsymbol{U}\right)$$

GR

$\alpha\,\psi^6\,W \times 4\pi\,r^2\,\rho\,v = -\dot{M}$

Steady-State Solutions (pre-shock)

$$\cancelto{0}{\partial_t \boldsymbol{U}} + \frac{1}{\sqrt{\gamma}} \, \partial_r \left[\alpha \sqrt{\gamma} \, \boldsymbol{F}^r \left(\boldsymbol{U} \right) \right] = \boldsymbol{S} \left(\boldsymbol{U} \right)$$

GR

$\alpha \, \psi^6 \, W \times 4\pi \, r^2 \, \rho \, v = -\dot{M}$

$\alpha \, h \, W = \mathcal{B}$

Steady-State Solutions (pre-shock)

$$\cancelto{0}{\partial_t \boldsymbol{U}} + \frac{1}{\sqrt{\gamma}} \, \partial_r \left[\alpha \sqrt{\gamma} \, \boldsymbol{F}^r \left(\boldsymbol{U} \right) \right] = \boldsymbol{S} \left(\boldsymbol{U} \right)$$

GR

$\alpha \, \psi^6 \, W \times 4\pi \, r^2 \, \rho \, v = -\dot{M}$

$\alpha \, h \, W = \mathcal{B}$

$p = K_1 \, \rho^\Gamma$

Steady-State Solutions (pre-shock)

$$\cancelto{0}{\partial_t \boldsymbol{U}} + \frac{1}{\sqrt{\gamma}} \, \partial_r \left[\alpha \sqrt{\gamma} \, \boldsymbol{F}^r \left(\boldsymbol{U} \right) \right] = \boldsymbol{S} \left(\boldsymbol{U} \right)$$

GR

$$\alpha \, \psi^6 \, W \times 4\pi \, r^2 \, \rho \, v = -\dot{M}$$

$$\alpha \, h \, W = \mathcal{B}$$

$$p = K_1 \, \rho^\Gamma$$

NR

Steady-State Solutions (pre-shock)

$$\cancelto{0}{\partial_t \boldsymbol{U}} + \frac{1}{\sqrt{\gamma}}\,\partial_r\left[\alpha\sqrt{\gamma}\,\boldsymbol{F}^r\left(\boldsymbol{U}\right)\right] = \boldsymbol{S}\left(\boldsymbol{U}\right)$$

GR

$$\alpha\,\psi^6\,W \times 4\pi\,r^2\,\rho\,v = -\dot{M}$$

$$\alpha\,h\,W = \mathcal{B}$$

$$p = K_1\,\rho^\Gamma$$

NR

$$4\pi\,r^2\,\rho\,v = -\dot{M}$$

Steady-State Solutions (pre-shock)

$$\cancelto{0}{\partial_t \boldsymbol{U}} + \frac{1}{\sqrt{\gamma}} \partial_r \left[\alpha \sqrt{\gamma} \, \boldsymbol{F}^r \left(\boldsymbol{U} \right) \right] = \boldsymbol{S} \left(\boldsymbol{U} \right)$$

GR

$$\alpha \, \psi^6 \, W \times 4\pi \, r^2 \, \rho \, v = -\dot{M}$$

$$\alpha \, h \, W = \mathcal{B}$$

$$p = K_1 \, \rho^\Gamma$$

NR

$$4\pi \, r^2 \, \rho \, v = -\dot{M}$$

$$\frac{1}{2} \, v^2 + h_{\mathrm{NR}} + \Phi = \mathcal{B}_{\mathrm{NR}}$$

Steady-State Solutions (pre-shock)

$$\cancelto{0}{\partial_t \boldsymbol{U}} + \frac{1}{\sqrt{\gamma}} \partial_r \left[\alpha \sqrt{\gamma} \, \boldsymbol{F}^r \left(\boldsymbol{U} \right) \right] = \boldsymbol{S} \left(\boldsymbol{U} \right)$$

GR

$\alpha \, \psi^6 \, W \times 4\pi \, r^2 \, \rho \, v = -\dot{M}$

$\alpha \, h \, W = \mathcal{B}$

$p = K_1 \, \rho^\Gamma$

NR

$4\pi \, r^2 \, \rho \, v = -\dot{M}$

$\frac{1}{2} \, v^2 + h_{\mathrm{NR}} + \Phi = \mathcal{B}_{\mathrm{NR}}$

$p = K_1 \, \rho^\Gamma$

Jump Conditions

$\boldsymbol{U}_1 \neq \boldsymbol{U}_2$

$\boldsymbol{F}^r(\boldsymbol{U}_1) = \boldsymbol{F}^r(\boldsymbol{U}_2)$

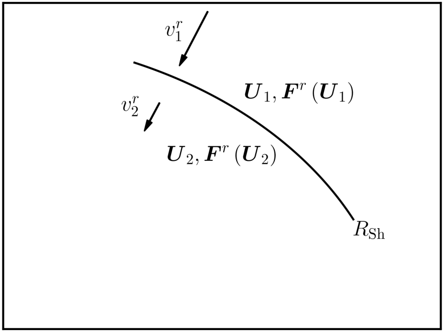
Yields:

ρ_2

$e_2\,(p_2)$

$K_2\,(> K_1)$

v_2^r



Steady-State Solutions (post-shock)

$$\cancelto{0}{\partial_t \boldsymbol{U}} + \frac{1}{\sqrt{\gamma}} \partial_r \left[\alpha \sqrt{\gamma} \, \boldsymbol{F}^r \left(\boldsymbol{U} \right) \right] = \boldsymbol{S} \left(\boldsymbol{U} \right)$$

GR

$$\alpha \, \psi^6 \, W \times 4\pi \, r^2 \, \rho \, v = -\dot{M}$$

$$\alpha \, h \, W = \mathcal{B}$$

$$p = K_2 \, \rho^\Gamma$$

NR

$$4\pi \, r^2 \, \rho \, v = -\dot{M}$$

$$\frac{1}{2} \, v^2 + h_{\mathrm{NR}} + \Phi = \mathcal{B}_{\mathrm{NR}}$$

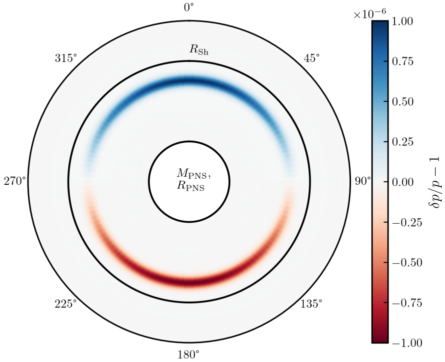
$$p = K_2 \, \rho^\Gamma$$

$$\eta(r) := \frac{r - R_{\mathrm{PNS}}}{R_{\mathrm{Sh}} - R_{\mathrm{PNS}}}$$

$$\frac{\delta p(\eta,\theta)}{p(\eta_{\mathrm{c}})} = 10^{-6} \times \exp\left[\frac{-(\eta - \eta_{\mathrm{c}})^2}{2\,\sigma^2}\right] \cos\theta$$

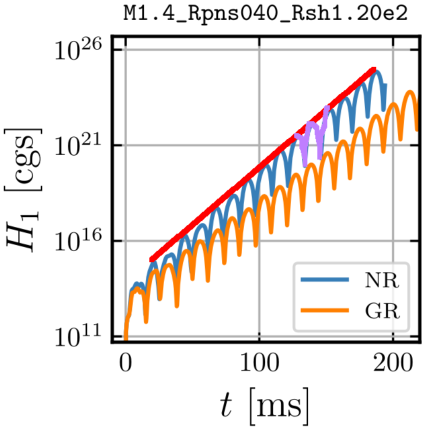
$$\eta_{\mathrm{c}} = 0.75$$

$$\sigma = 0.05$$



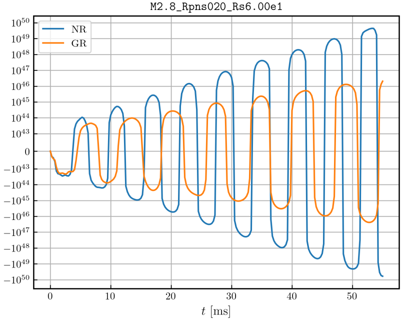
$$F\left(t\right) = F\left(0\right)\,e^{2\,\omega\,t}\,\sin^2\left(\frac{2\pi\,t}{T} + \delta\right)$$

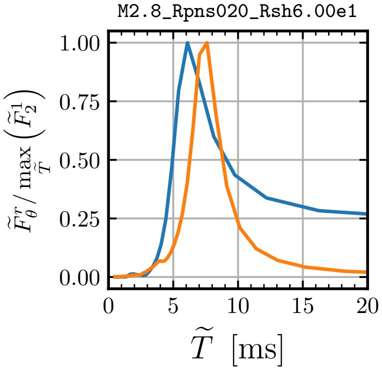
(Blondin and Mezzacappa, 2006)



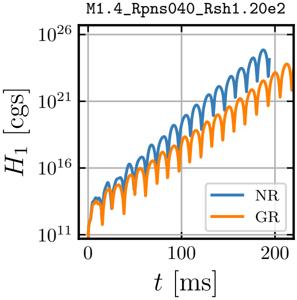
$$F_\theta^r := \alpha\,\psi^6\,h\,W^2 \times \sqrt{\bar{\gamma}}\,\rho\,v^r\,v_\theta$$

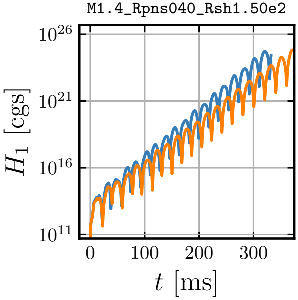
$$\widetilde{F}_\theta^r := \mathrm{FFT}\left\{F_\theta^r\right\}$$

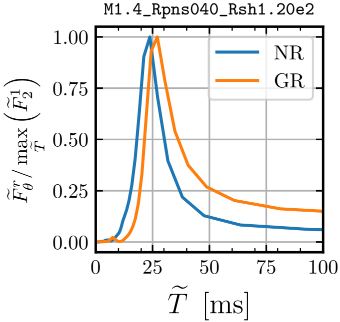


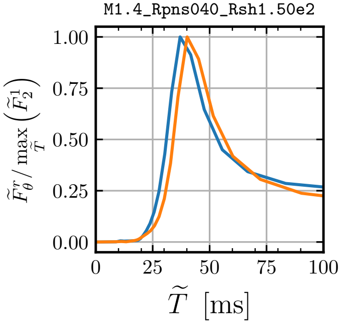


T defined as the unique $\widetilde{T}$ such that $\widetilde{F}_\theta^r\left(\widetilde{T}\right) = 1$


M1.4_Rpns040_Rsh1.20e2
H_1 [cgs]
10^{26}
10^{21}
10^{16}
10^{11}
0
100
200
t [ms]
NR
GR


M1.4_Rpns040_Rsh1.50e2
H_1 [cgs]
10^{26}
10^{21}
10^{16}
10^{11}
0
100
200
300
t [ms]


M1.4_Rpns040_Rsh1.20e2
$\widetilde{F}_\theta^r / \max_{\widetilde{T}} \left(\widetilde{F}_2^1\right)$
1.00
0.75
0.50
0.25
0.00
0
25
50
75
100
$\widetilde{T}$ [ms]
NR
GR


M1.4_Rpns040_Rsh1.50e2
$\widetilde{F}_\theta^r / \max_{\widetilde{T}} \left(\widetilde{F}_2^1\right)$
1.00
0.75
0.50
0.25
0.00
0
25
50
75
100
$\widetilde{T}$ [ms]