thornado-Hydro (xCFC)

Samuel J. Dunham

October 18, 2023

Brandon Barker (MSU/LANL), Jesse Buffaloe (UTK),
SJD (Vanderbilt,UTK), Zack Elledge (UTK),
Eirik Endeve (ORNL/UTK), Anthony Mezzacappa (UTK),
David Pochik (OSU), Nick Roberts (UTK)

thornado

https://github.com/endeve/thornado

My Website

https://www.samueljdunham.com

thornado

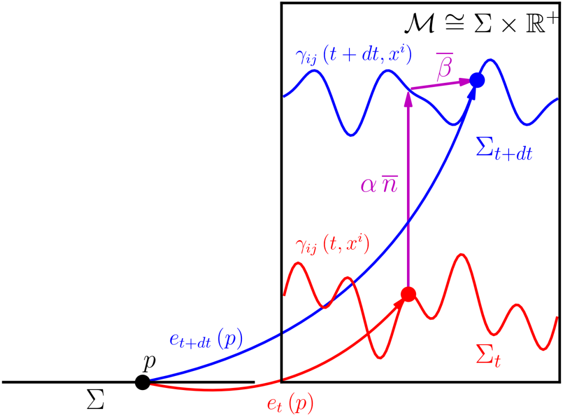
toolkit for high-order neutrino-radiation hydrodynamics

- DG
- SSPRK/IMEX
- GR (xCFC)
- Hydro[a] (Valencia)
- Neutrino transport[b] (M1)
- Interfaces to tabulated EoS/Opacities (weaklib: `https://github.com/starkiller-astro/weaklib`)

- Fluid self-gravity via Poseidon: `https://github.com/jrober50/Poseidon`
- GPUs via OpenACC or OpenMP pragmas
- MPI parallelism and AMR via AMReX: `https://github.com/AMReX-Codes/amrex`

[a] Endeve et al. (2019); Dunham et al. (2020); Pochik et al. (2021)
[b] Laiu et al. (2021)

3+1 Decomposition



$$ds^2 = g_{\mu\nu}\,dx^\mu\,dx^\nu = -\alpha^2\,dt^2 + \gamma_{ij}\left(dx^i + \beta^i\,dt\right)\left(dx^j + \beta^j\,dt\right)$$

Conformally-Flat Condition

Developed by Wilson et al. (1996), extended by Cordero-Carrión et al. (2009)

$$\gamma_{ij}\left(x\right) = \psi^{4}\left(x\right)\overline{\gamma}_{ij}\left(x^{i}\right)$$

$$K = 0,\ \partial_{t} K = 0$$

$$\text{(Always and everywhere)}$$

- Exact in spherical symmetry!
- Hyperbolic $\rightarrow$ Elliptic equations
- Good for long-time simulations

Special case: Schwarzchild spacetime in isotropic coordinates

$$\alpha\left(r\right) = \left(1 + R_{\mathrm{Sc}}/r\right)\left(1 - R_{\mathrm{Sc}}/r\right)^{-1}$$

$$\psi\left(r\right) = 1 + R_{\mathrm{Sc}}/r$$

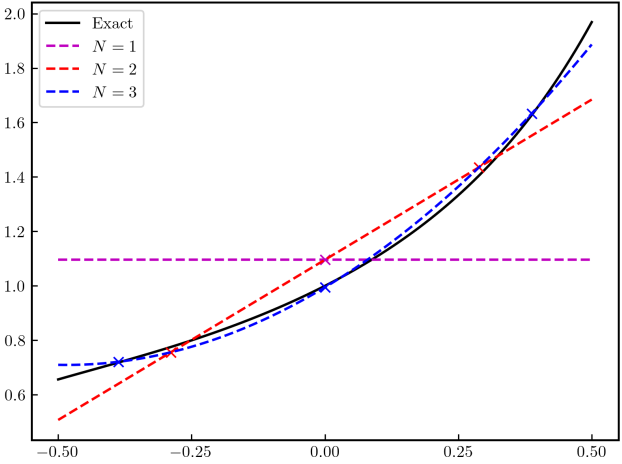
$$\beta^{i}\left(r\right) = 0,$$

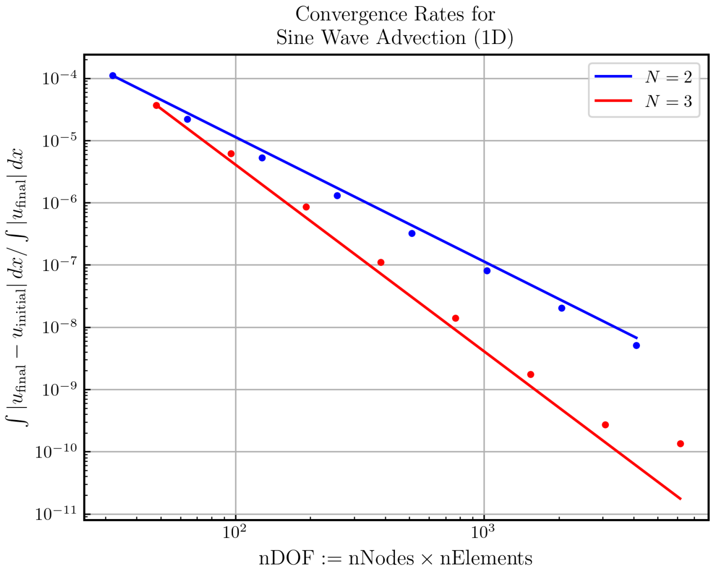
with

$$r > R_{\mathrm{Sc}} := M/2$$

Discontinuous Galerkin (DG)

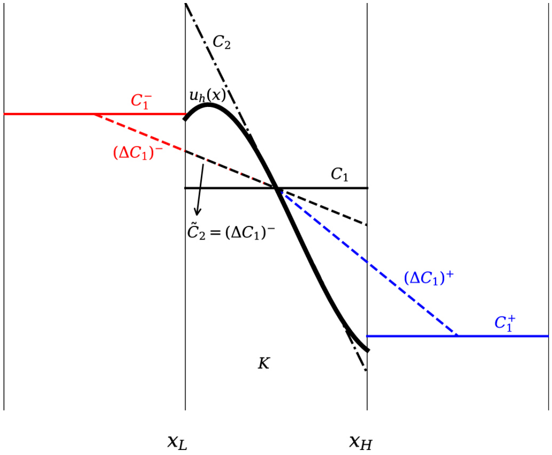
$$u_h\left(x,t\right) := \sum_{i=1}^{N} u_i\left(t\right)\ \ell_i\left(x\right)$$





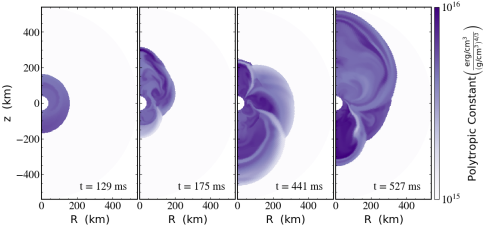
Convergence Rates for
Sine Wave Advection (1D)
$N = 2$
$N = 3$
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
$\int |u_{\mathrm{final}} - u_{\mathrm{initial}}|\, dx / \int |u_{\mathrm{final}}|\, dx$
10^{2}
10^{3}
nDOF := nNodes × nElements

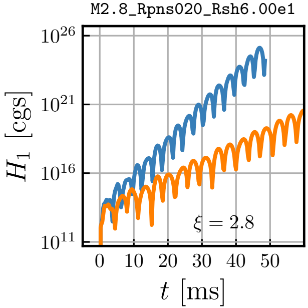
$$u_h(x,t) = \sum_{n=1}^{N} C_n(t)\; P_n(x) \implies \tilde{u}_h(x,t) = C_1(t)\; P_1(x) + \tilde{C}_2(t)\; P_2(x)$$



Standing Accretion Shock Instability

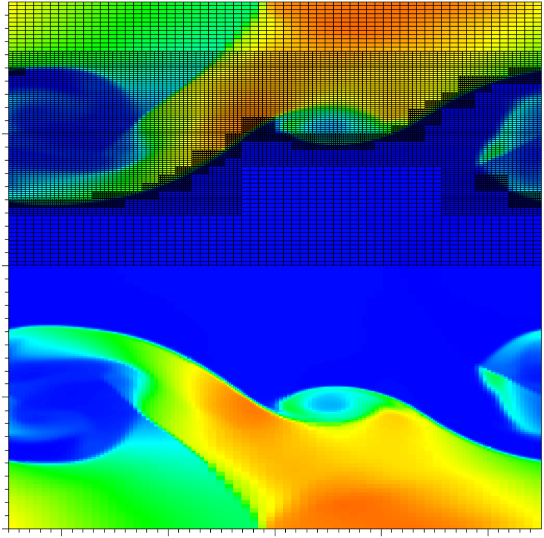
Used thornado to investigate the role of GR on the SASI[1]



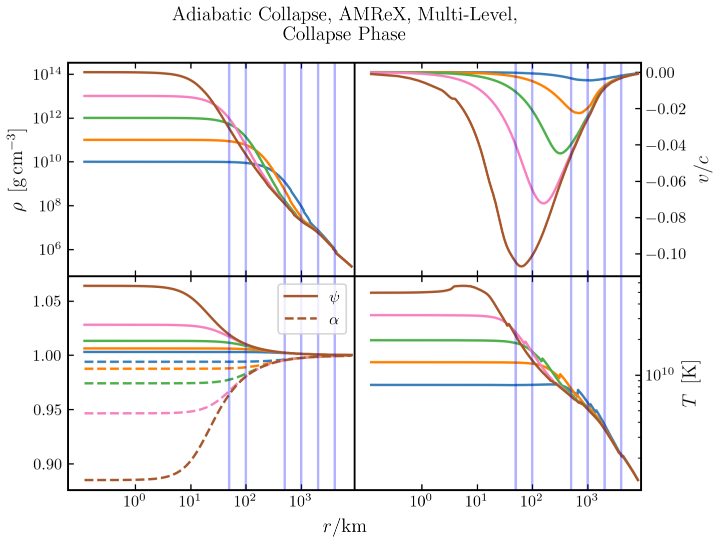


[1]Dunham et al. (2020, 2023)

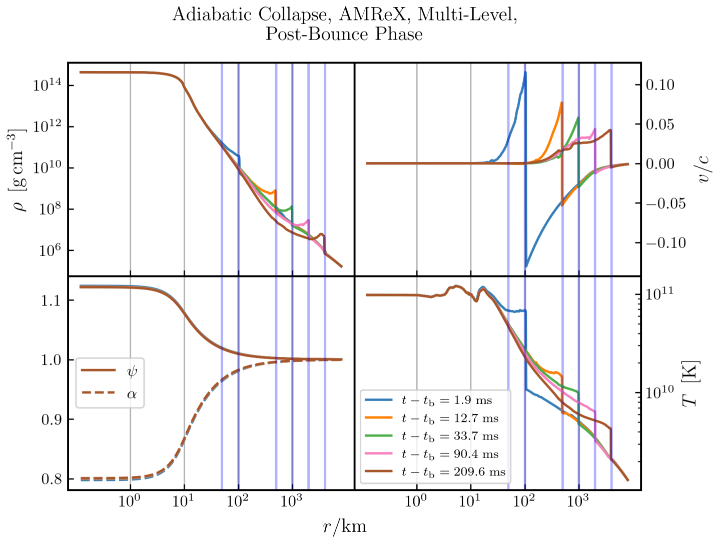
Kelvin–Helmholtz Instability



Adiabatic Collapse (SMR)



Adiabatic Collapse (SMR)



Bibliography

Eirik Endeve, Jesse Buffaloe, Samuel J. Dunham, Nick Roberts, Kristopher Andrew, Brandon Barker, David Pochik, Juliana Pulsinelli, and Anthony Mezzacappa. thornado-hydro: towards discontinuous Galerkin methods for supernova hydrodynamics. In *Journal of Physics Conference Series*, volume 1225 of *Journal of Physics Conference Series*, page 012014, May 2019. doi: 10.1088/1742-6596/1225/1/012014.

Samuel J. Dunham, E. Endeve, A. Mezzacappa, J. Buffaloe, and K. Holley-Bockelmann. A discontinuous Galerkin method for general relativistic hydrodynamics in thornado. In *Journal of Physics Conference Series*, volume 1623 of *Journal of Physics Conference Series*, page 012012, September 2020. doi: 10.1088/1742-6596/1623/1/012012.

David Pochik, Brandon L. Barker, Eirik Endeve, Jesse Buffaloe, Samuel J. Dunham, Nick Roberts, and Anthony Mezzacappa. thornado-hydro: A Discontinuous Galerkin Method for Supernova Hydrodynamics with Nuclear Equations of State. ApJS, 253(1):21, March 2021. doi: 10.3847/1538-4365/abd700.

M. Paul Laiu, Eirik Endeve, Ran Chu, J. Austin Harris, and O. E. Bronson Messer. A dg-imex method for two-moment neutrino transport: Nonlinear solvers for neutrino–matter coupling*. *The Astrophysical Journal Supplement Series*, 253(2):52, apr 2021. doi: 10.3847/1538-4365/abe2a8. URL `https://dx.doi.org/10.3847/1538-4365/abe2a8`.

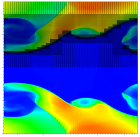
J. R. Wilson, G. J. Mathews, and P. Marronetti. Relativistic numerical model for close neutron-star binaries. Phys. Rev. D, 54(2):1317–1331, July 1996. doi: 10.1103/PhysRevD.54.1317.

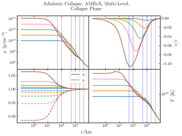
Isabel Cordero-Carrión, Pablo Cerdá-Durán, Harald Dimmelmeier, José Luis Jaramillo, Jérôme Novak, and Eric Gourgoulhon. Improved constrained scheme for the Einstein equations: An approach to the uniqueness issue. Phys. Rev. D, 79(2):024017, January 2009. doi: 10.1103/PhysRevD.79.024017.

Samuel J. Dunham, Eirik Endeve, Anthony Mezzacappa, John M. Blondin, Jesse Buffaloe, and Kelly Holley-Bockelmann. A Parametric Study of the SASI Comparing General Relativistic and Non-Relativistic Treatments. *arXiv e-prints*, art. arXiv:2307.10904, July 2023. doi: 10.48550/arXiv.2307.10904.

Summary

Can run pure hydro problems in GR with AMR





Doing GR, adiabatic collapse simulations with mesh refinement

Working on coupling GR transport to existing hydro+gravity modules